FLOWS IN PIPES AND PUMPS

Simple Pipe Systems

The head losses equations are repeated here for convenience:

$$h_L = f\frac{L}{D}\frac{V^2}{2g}$$

$$h_L = K\frac{V^2}{2g}$$

Since use will be made of those concepts, the reader should review those sections in their entirety before proceeding. Figure 7.10 may be utilized to determine the friction factor. In addition to the Darcy– Weisbach formulation, the Hazen–Williams formula has found wide use among practitioners. It is

$$h_L = \frac{K_1 L}{C^{1.85} D^{4.87}} Q^{1.85}$$

where Q is discharge
L is length of the pipe element
C is coefficient dependent on the pipe roughness
K1 is 10.59 (for SI units) and 4.72 (for English units); note that K1 depends on the system of units

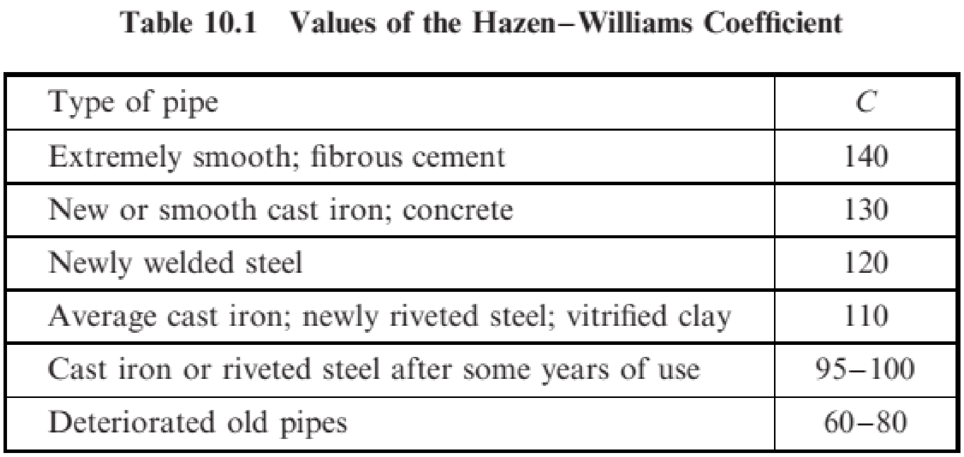
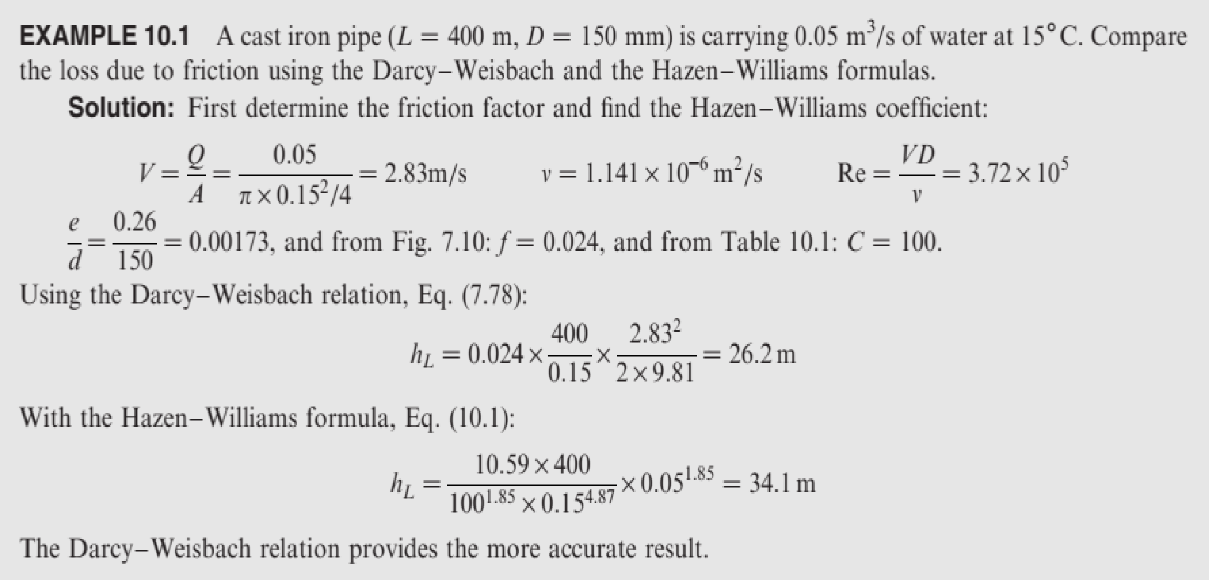


Table 10.1 Values of the Hazen–Williams Coefficient

Type of pipe	C
Extremely smooth; fibrous cement	140
New or smooth cast iron; concrete	130
Newly welded steel	120
Average cast iron; newly riveted steel; vitrified clay	110
Cast iron or riveted steel after some years of use	95–100
Deteriorated old pipes	60–80

The Hazen–Williams loss formula is empirically based, and is less accurate than the Darcy–Weisbach relation.



EXAMPLE 10.1 A cast iron pipe ($L = 400$ m, $D = 150$ mm) is carrying 0.05 m^3/s of water at 15°C. Compare the loss due to friction using the Darcy–Weisbach and the Hazen–Williams formulas.

Solution: First determine the friction factor and find the Hazen–Williams coefficient:

$$V = \frac{Q}{A} = \frac{0.05}{\pi \times 0.15^2/4} = 2.83\,\text{m/s} \qquad \nu = 1.141 \times 10^{-6}\,\text{m}^2/\text{s} \qquad \text{Re} = \frac{VD}{\nu} = 3.72 \times 10^5$$

$\frac{e}{d} = \frac{0.26}{150} = 0.00173$, and from Fig. 7.10: $f = 0.024$, and from Table 10.1: $C = 100$.

Using the Darcy–Weisbach relation, Eq. (7.78):

$$h_L = 0.024 \times \frac{400}{0.15} \times \frac{2.83^2}{2 \times 9.81} = 26.2\,\text{m}$$

With the Hazen–Williams formula, Eq. (10.1):

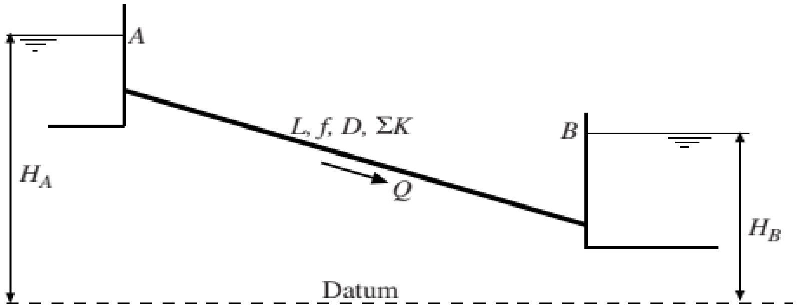
$$h_L = \frac{10.59 \times 400}{100^{1.85} \times 0.15^{4.87}} \times 0.05^{1.85} = 34.1\,\text{m}$$

The Darcy–Weisbach relation provides the more accurate result.

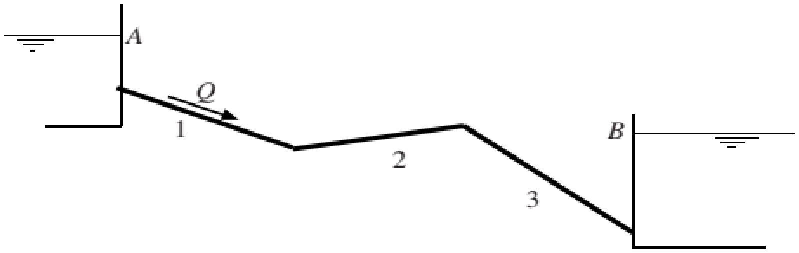
Hydraulics of Simple Pipe Systems

The energy and continuity equations are employed to analyze pipe systems. Normally, the predicted parameters are discharge Q and piezometric head $H = p/\gamma + z$. The kinetic energy term is negligible compared to the magnitude of the hydraulic grade line, that is, $V^2/2g \ll p/\gamma + z$. Referring to Fig. 10.1(a), the energy equation for a single reach of pipe is

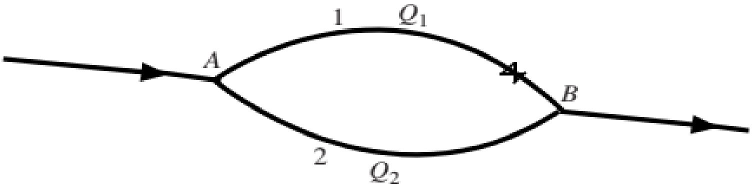
$$H_A - H_B = \sum h_f = \left(f\frac{L}{D}\frac{1}{2gA^2} + \frac{\Sigma K}{2gA^2}\right)Q^2$$



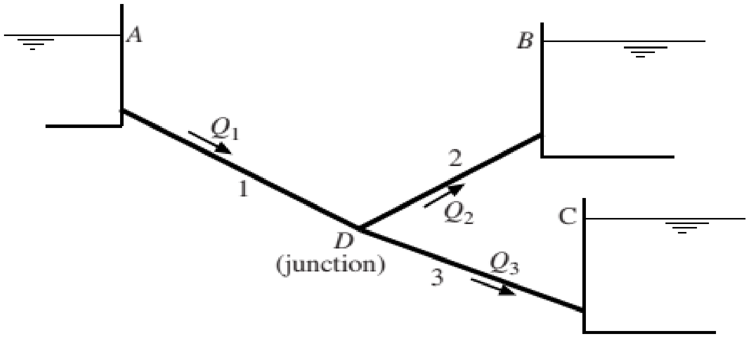
(*a*) Single element



(*b*) Three elements in series



(*c*) Two parallel elements



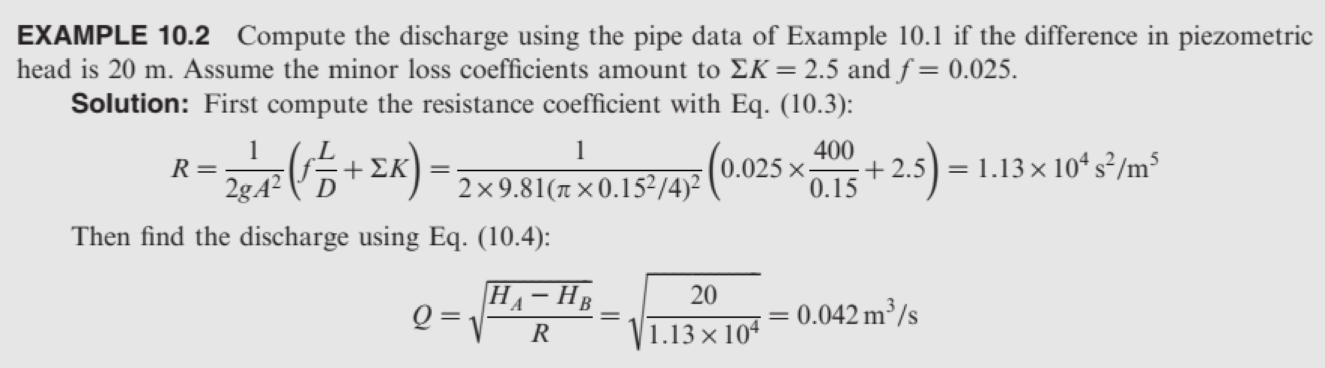
(*d*) Three branch elements

Figure 10.1 Simple pipe systems.

If we let the friction and minor loss terms be represented by a resistant, or loss, coefficient R, defined as

$$R = \frac{1}{2gA^2}\left(f\frac{L}{D} + \sum K\right)$$

$$H_A - H_B = RQ^2$$



EXAMPLE 10.2 Compute the discharge using the pipe data of Example 10.1 if the difference in piezometric head is 20 m. Assume the minor loss coefficients amount to $\Sigma K = 2.5$ and $f = 0.025$.

Solution: First compute the resistance coefficient with Eq. (10.3):

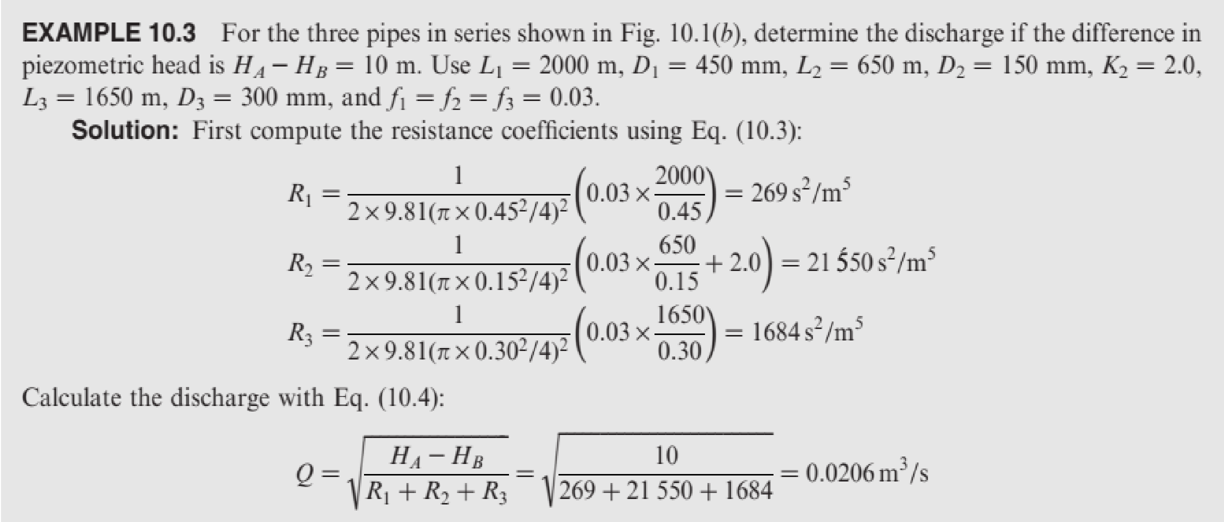
$$R = \frac{1}{2gA^2}\left(f\frac{L}{D} + \Sigma K\right) = \frac{1}{2 \times 9.81(\pi \times 0.15^2/4)^2}\left(0.025 \times \frac{400}{0.15} + 2.5\right) = 1.13 \times 10^4\ \mathrm{s^2/m^5}$$

Then find the discharge using Eq. (10.4):

$$Q = \sqrt{\frac{H_A - H_B}{R}} = \sqrt{\frac{20}{1.13 \times 10^4}} = 0.042\ \mathrm{m^3/s}$$

Figure 10.1(b) shows a series pipe system consisting of three reaches, each with a specified loss coefficient. Since the same discharge Q exists in each reach, the energy equation from location A to location B is

$$H_A - H_B = R_1Q^2 + R_2Q^2 + R_3Q^2 = (R_1 + R_2 + R_3)Q^2$$



EXAMPLE 10.3 For the three pipes in series shown in Fig. 10.1(*b*), determine the discharge if the difference in piezometric head is $H_A - H_B = 10$ m. Use $L_1 = 2000$ m, $D_1 = 450$ mm, $L_2 = 650$ m, $D_2 = 150$ mm, $K_2 = 2.0$, $L_3 = 1650$ m, $D_3 = 300$ mm, and $f_1 = f_2 = f_3 = 0.03$.

Solution: First compute the resistance coefficients using Eq. (10.3):

$$R_1 = \frac{1}{2 \times 9.81(\pi \times 0.45^2/4)^2}\left(0.03 \times \frac{2000}{0.45}\right) = 269\ \mathrm{s^2/m^5}$$

$$R_2 = \frac{1}{2 \times 9.81(\pi \times 0.15^2/4)^2}\left(0.03 \times \frac{650}{0.15} + 2.0\right) = 21\,550\ \mathrm{s^2/m^5}$$

$$R_3 = \frac{1}{2 \times 9.81(\pi \times 0.30^2/4)^2}\left(0.03 \times \frac{1650}{0.30}\right) = 1684\ \mathrm{s^2/m^5}$$

Calculate the discharge with Eq. (10.4):

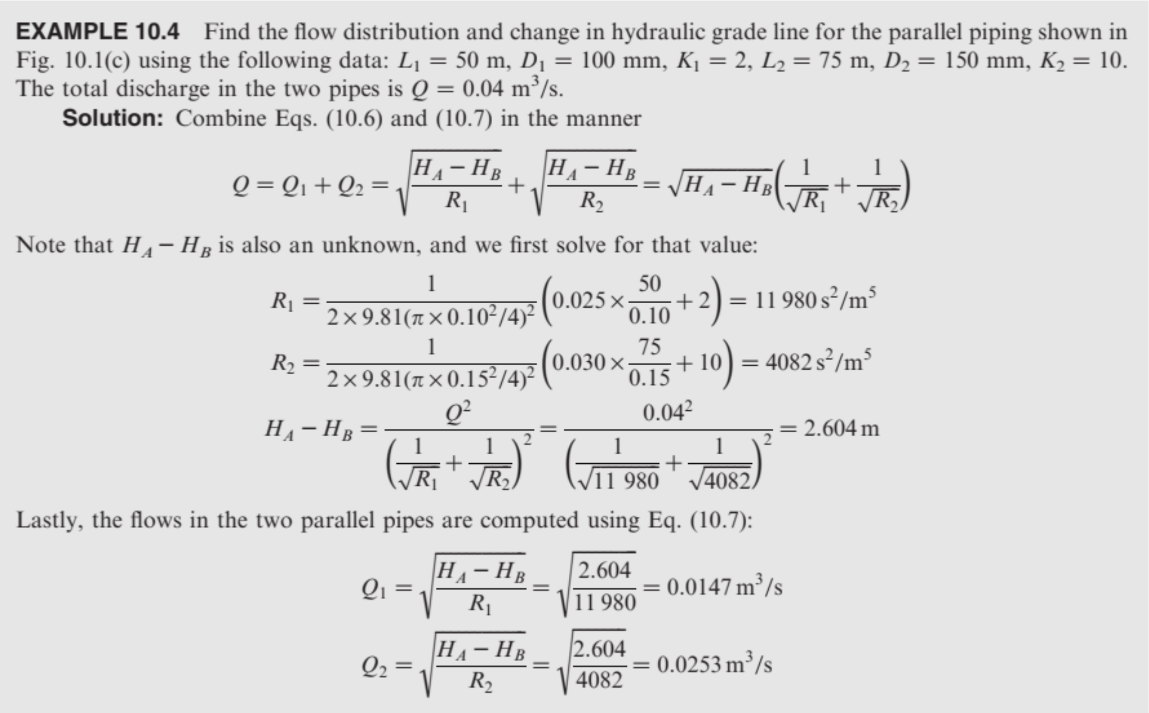
$$Q = \sqrt{\frac{H_A - H_B}{R_1 + R_2 + R_3}} = \sqrt{\frac{10}{269 + 21\,550 + 1684}} = 0.0206\ \mathrm{m^3/s}$$

Parallel piping is illustrated in Fig. 10.1(c); even though only two pipes are shown, any number of pipes can be placed in parallel. The pipes are joined at locations A and B, and each pipe has its own unique geometry and minor loss term. The continuity balance at either junction A or B requires that

$$Q = Q_1 + Q_2$$

For the two pipe elements, the required energy equations from location A to B are

$$H_A - H_B = R_1 Q_1^2$$
$$H_A - H_B = R_2 Q_2^2$$



EXAMPLE 10.4 Find the flow distribution and change in hydraulic grade line for the parallel piping shown in Fig. 10.1(c) using the following data: $L_1 = 50$ m, $D_1 = 100$ mm, $K_1 = 2$, $L_2 = 75$ m, $D_2 = 150$ mm, $K_2 = 10$. The total discharge in the two pipes is $Q = 0.04\ \text{m}^3/\text{s}$.

Solution: Combine Eqs. (10.6) and (10.7) in the manner

$$Q = Q_1 + Q_2 = \sqrt{\frac{H_A - H_B}{R_1}} + \sqrt{\frac{H_A - H_B}{R_2}} = \sqrt{H_A - H_B}\left(\frac{1}{\sqrt{R_1}} + \frac{1}{\sqrt{R_2}}\right)$$

Note that $H_A - H_B$ is also an unknown, and we first solve for that value:

$$R_1 = \frac{1}{2 \times 9.81(\pi \times 0.10^2/4)^2}\left(0.025 \times \frac{50}{0.10} + 2\right) = 11\,980\ \text{s}^2/\text{m}^5$$

$$R_2 = \frac{1}{2 \times 9.81(\pi \times 0.15^2/4)^2}\left(0.030 \times \frac{75}{0.15} + 10\right) = 4082\ \text{s}^2/\text{m}^5$$

$$H_A - H_B = \frac{Q^2}{\left(\frac{1}{\sqrt{R_1}} + \frac{1}{\sqrt{R_2}}\right)^2} = \frac{0.04^2}{\left(\frac{1}{\sqrt{11\,980}} + \frac{1}{\sqrt{4082}}\right)^2} = 2.604\ \text{m}$$

Lastly, the flows in the two parallel pipes are computed using Eq. (10.7):

$$Q_1 = \sqrt{\frac{H_A - H_B}{R_1}} = \sqrt{\frac{2.604}{11\,980}} = 0.0147\ \text{m}^3/\text{s}$$

$$Q_2 = \sqrt{\frac{H_A - H_B}{R_2}} = \sqrt{\frac{2.604}{4082}} = 0.0253\ \text{m}^3/\text{s}$$

An example of branch piping is illustrated in Fig. 10.1(d); it is made up of three elements connected to a single junction. Typically, the piezometric heads at locations A to C are considered to be known, and the unknowns are the discharges Q1, Q2, and Q3 in each line and the piezometric head at location D.

Analysis proceeds by assuming the direction of flow and writing the energy balance in each element:

$$H_A - H_D = R_1 Q_1^2$$

$$H_D - H_B = R_2 Q_2^2$$

$$H_D - H_C = R_3 Q_3^2$$

$$Q_1 - Q_2 - Q_3 = 0$$

Note that a flow direction was assumed in each pipe. One method of solution is as follows:

1. Assume HD at the junction.
2. Compute Q1, Q2, and Q3 in the three branches.
3. Substitute Q1, Q2, and Q3 into continuity equation to check for continuity balance. Generally, the flow imbalance ΔQ = Q1 - Q2 - Q3 will be nonzero at the junction.
4. Adjust the head HD and repeat steps 2 and 3 until DQ is within desired limits. It may be necessary to correct the sign in one or more of the equations if during the iterations HD moves from above or below one of the reservoirs or vice versa.

EXAMPLE 10.5 Determine the flow rates and the piezometric head at the junction for the branch system of Fig. 10.1(*d*). Assume constant friction factors. $H_A = 12$ m, $H_B = 15$ m, $H_C = 5$ m, $f_1 = f_2 = f_3 = 0.02$, $L_1 = 200$ m, $D_1 = 100$ mm, $L_2 = 150$ m, $D_2 = 100$ mm, $L_3 = 750$ m, $D_3 = 150$ mm.

Solution: Use the four-step procedure outlined above. First compute the resistance coefficients using Eq. (10.3); the result is $R_1 = 33\,890\ \mathrm{s^2/m^5}$, $R_2 = 27\,280\ \mathrm{s^2/m^5}$, and $R_3 = 16\,570\ \mathrm{s^2/m^5}$. We assume that H_D is lower than H_A and H_B, but higher than H_C. Consequently, the resulting flow directions are Q_1 from *A* to *D*, Q_2 from *B* to *D*, and Q_3 from *D* to *C*. An iterative solution is shown in the accompanying table. Iteration ceases when $|\Delta Q| < 0.001$ units.

Iteration	H_D (assumed)	Q_1 (Eq. (10.8))	Q_2 (Eq. (10.10))	Q_3 (Eq. (10.9))	$Q_1 + Q_2 - Q_3$ (Eq. (10.11))
1	12	0	0.01049	0.02055	−0.01006
2	11	0.00543	0.01211	0.01903	−0.00149
3	10	0.00768	0.01354	0.01737	+0.00385
4	10.74	0.00610	0.01250	0.01861	−0.00002

Hence the resulting solution is $H_D = 10.7$ m, $Q_1 = 0.0061\ \mathrm{m^3/s}$, $Q_2 = 0.0125\ \mathrm{m^3/s}$, and $Q_3 = 0.0186\ \mathrm{m^3/s}$.